4 The Rhumb Line and the Great Circle in Navigation

4.1 Details on Great Circles

In *fig. GN 4.1* two Great Circle/Rhumb Line cases are shown, one in each hemisphere. In each case the shorter distance between any 2 points will be via the Great Circle route.

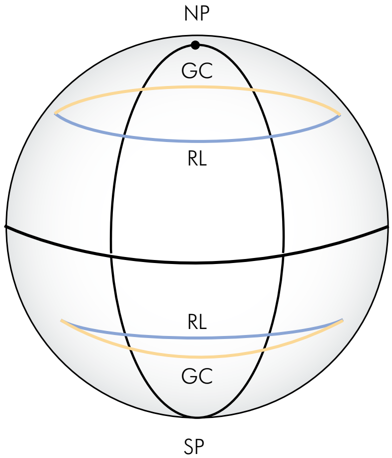


Fig. GN 4.1 Locations of GC and RL in the northern and in the southern hemisphere

When the two points concerned are separated by 180° of longitude. i.e., they lie on a meridian and its associated anti-meridian, e.g. 000°E/W and 180° E/W. In this case the Great Circle will follow the meridian until it reaches the closer pole and then the anti-meridian until reaching the second point. It is easy to calculate this Great Circle distance.

In *table GN 4.1* some figures are given for comparing the Great Circle and the Rhumb Line tracks between place of departure at 6000N 01000E and different destinations, all of which are located on the 6000N parallel. In this case the Rhumb Line track will be 270°, and the Rhumb Line distance is easy to calculate, using the formula for departure.

Observing the Great Circle track and the rhumb line track between the same positions on the Earth's surface:

- The Great Circle track will continuously change it's true direction, while the

DEP	DEST	DLONG	RLDIST	GCDIST	DIFF NM	DIST %	Initial GCTT
01000 E	01000 W	20	600	597.7	2.3	0.4	278.7
01000 E	03000 W	40	1200	1181.6	18.4	1.5	287.5
01000 E	05000 W	60	1800	1737.3	62.6	3.5	296.6
01000 E	11000 W	120	3600	3079.1	520.9	14.5	326.3

Table GN 4.1

rhumb line track by definition has a fixed true direction

- The Great Circle track will be located between the Rhumb Line track and the nearest Pole
- The Great Circle track will be the shortest of the two, and the difference in distance between the two tracks will increase with latitude and with change of longitude between the two positions.

General calculation of great circle directions and distances are today made using hand held electronic calculators, or navigation calculators built into navigation equipment such as RNAV, GPS and INS.

Fig. GN 4.2 shows a view of the Earth as seen from a point above the Equator. A Great Circle is passing through the position on the Equator directly below the viewer at 100°00'W. The Great Circle has an inclination of 33° as it crosses the Equator, which is an angle of 57° to the meridian. From this perspective it is easy to see that the Great Circle, on its path northeasterly from the Equator, will reach higher latitudes. The highest latitude reached will be the same as the inclination to the Equator i.e. 33°00'N.

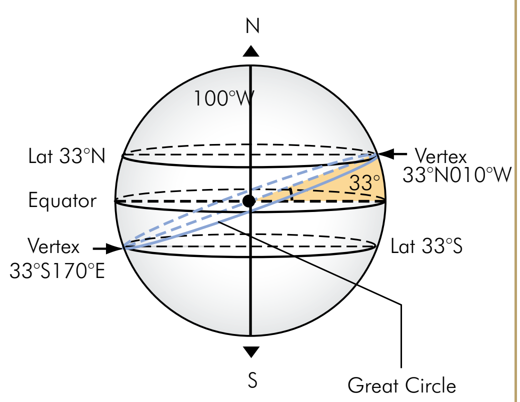


Fig. GN 4.2 The Vertex of a Great Circle

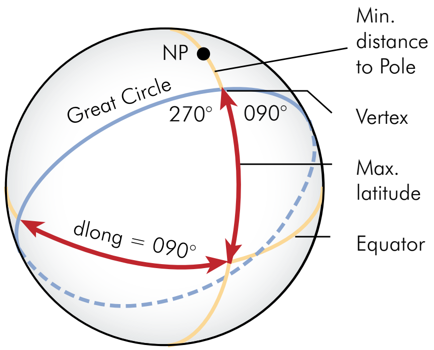
It will also be appreciated that the longitude of the Great Circle, where it reaches its highest latitude, will be 90° different from the longitude of the position where it crosses the Equator, in this case 010°00'W. It will then proceed back towards the Equator crossing it 19° removed from the highest point i.e. 080°00'E. The Great Circle then continues to the highest southern latitude of 33°00'S at 170°00'E.

Vertex

The position where the Great Circle reaches its highest latitude is called the **vertex** of that Great Circle. *Fig. GN 4.3* shows a Great Circle with its Northern Hemisphere vertex. In order not to complicate the sketch, a broken line indicates the other half of the Great Circle (running on the far side of the Earth).

Every great circle has two vertices; one in the Northern and one in the Southern Hemisphere.

At the vertex the direction of the Great Circle is 090° or 270°, and it is at this point closest to the nearest Pole, and furthest from the Equator. Meridians and the Equator are Great Circles that have special



The position where the Great Circle passes closest to the pole is called Vertex.

At the Vertex the direction of the Great Circle is 090° or 270°.

At the Vertex the Great Circle will achieve its highest latitude.

There is also a similar Vertex point on the southern hemisphere.

The longitude of the Vertex is always 090° different from the longitude where the Great Circle passes the Equator.

Fig. GN 4.3 A Great Circle passing the Equator

characteristics. They may be considered to have no vertices, *see fig. GN 4.3.*

As an example some satellites orbit the Earth in circles. These may be regarded as Great Circles, and the paths of the satellite, projected down onto the Earth's surface, may also be regarded as great circles. It will therefore be evident that the satellite's orbit will not reach a higher latitude than the inclination it has with the Equator.

When considering the orbital paths of satellites, it is well to remember that they are following a path in space. A circular orbit will retain its circular shape but, as the Earth rotates under the satellite at the same time, the track of the satellite on the surface of the Earth will not be a Great Circle.

However, the northern and southern limits of the satellite track will still be given by the inclination at the Equator passing.

Summary on Great Circles

- A Great Circle has it's vertex where it reaches it highest latitude
- At the Vertex the Great Circle true direction is 090°/270°, running East – West
- A Great Circle will have 2 vertices, one on the northern and one on the southern hemisphere
- The vertices will be located with dlong 090° from the positions where the Great Circle cuts the Equator
- The latitudes of the vertices will equal the inclination between the plane of Equator and the plane of the Great Circle.

Example

A satellite has an area of coverage on the surface of the Earth that is 600 NM wide. The most northerly position to be covered by this satellite is 72°00'N. What should the inclination with the Equator be?

Solution

The satellite will at any time cover a circle with radius 300 NM on the surface of the Earth. This means that the satellite track may be 300 NM south of its most northerly point of coverage. 300 NM south equals 300 NM/60 = 5° of latitude.

The Vertex must be at
72°00'N - 5° = 67°00'N.

The inclination at Equator must be the same as latitude of the vertex, 67°.

(An inclination of 67° will result in the satellite coverage just touching 72°00'N on every orbit, and the period this latitude is covered will be very short. Better coverage at 72°00'N will be possible by increasing the inclination).

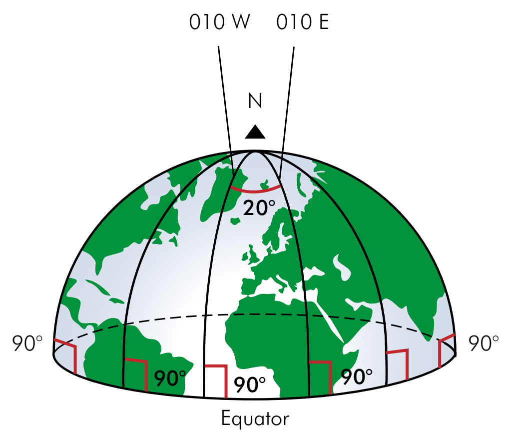
4.2 Earth Convergency

4.2.1 Earth Convergency of the Meridians

Fig. GN 4.4 shows a part of the Northern Hemisphere. If the meridians 01000W and 01000E are considered, it is clear that they both cross the Equator at right angles, and consequently are parallel when they cross the Equator. When these meridians meet, or intercept, at the poles, they intercept with an angle of 20 degrees. This angle is the same value as the dlong between them.

This is an important finding, due to the fact that the Earth is a spherical surface, as we travel between the Equator and the poles, two meridians change directions with an amount dependant on the difference of longitude between them.

In the case above we could say that the convergency of the meridians is zero at the Equator and increasing to 20°, or dlong, at the poles.



All Meridians are parallel as they pass the Equator.

All Meridians meet at the poles with an angle equal to dlong.

Fig. GN 4.4 Earth convergency

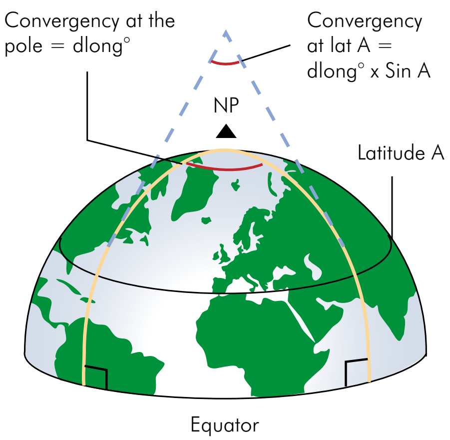
The change of the direction we have seen is obviously increasing with increased latitude, but the change is not linear, but a function of the sine of the latitude and dlong.

Therefore we can derive a formula for Earth Convergency:

$$\text{Earth convergency} = \text{dlong}^\circ \times \sin(\text{lat})$$
$$= \Delta\lambda \times \sin\varphi$$

Definition
Earth Convergency is the angle of inclination of meridians towards one another.

In *fig. GN 4.5* the most important facts on Earth Convergency are shown. The



Convergency at the Equator = 0°
Convergency at the Pole = dlong° (Δλ)
The general equation for calculating Earth convergency is:

Earth conv = dlong° x sin (lat)

(Earth conv = $\Delta\lambda \times \sin \varphi$)

Fig. GN 4.5 Facts on Earth convergency

consequences of these facts will be discussed in the chapter on Charts.

Earth Convergency

- Is the difference in direction between two meridians
- Increases with an increase in latitude
- Increases with an increase in longitude
- Is zero at the Equator and is equal to dlong at the poles
- May be calculated at any latitude by using the equation:

Earth convergency = dlong° x sin (lat)

$= \Delta\lambda \times \sin \varphi$

- Earth Convergency between two positions at different latitudes, may be calculated to an approximate value substituting "lat" by "mean lat" in the equation above.

4.2.2 Earth Convergency Calculations

A simple formula may be used to calculate the convergency between two meridians:

Earth convergency = dlong° x sin (mean lat)

$= \Delta\lambda \times \sin \varphi_m$

(Only use the above given formula when: $\Delta\lambda < 090°$ and the distance < 1000 NM.)

As mentioned above, the dlong has to be entered in the formula as degrees and decimals.

The formula is only correct for one value of latitude. If we require to find the Earth Convergency between the meridians running through two positions at different latitude, mean latitude between the positions must be substituted for Lat in the formula. In this case the formula is no longer accurate, and great care should be taken using it. Generally the inaccuracies increase with higher latitudes and higher values of dlong. The reason for this is that sine to an angle is not a linear function, the arithmetic mean value of lat is therefore not correctly used.

The Earth Convergency of meridians may also be evaluated by observing the direction of Great Circles. It is equivalent to the angular change of direction of a great circle course as it passes from one meridian to another.

Using this technique we may find the exact value of Earth Convergency between two positions even if they are at different latitudes and at different longitudes.

$GCTT_A$ is an abbreviation for "Great Circle True Track Initial", and $GCTT_B$ stands for "Great Circle True Track Final". Even if the term "Track" is used in these abbreviations, the short terms are often used also for Great Circles representing types of line other than tracks, even if the correct procedure would then be to use "GCT_A" and "GCT_B". Other expressions may be encountered.

In *fig. GN 4.6* the positions A and B, situated in the Northern Hemisphere, are indicated as well as the meridians running through these positions. The Great Circle running through A and B is shown. Although the diagram may not accurately represent the directions, it does illustrate that the true direction of the Great Circle, as it passes through A, (labelled $GCTT_A$ on the sketch), is of a lower value than the true direction of the Great Circle as it passes B.

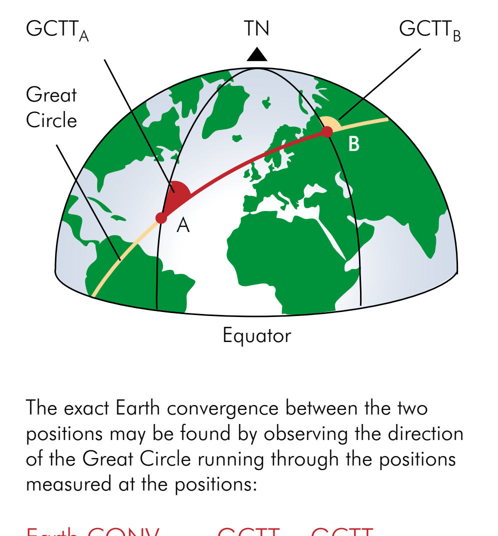


Fig. GN 4.6 Earth convergence between two positions on the Northern Hemisphere

The direction at B is labelled $GCTT_B$. Using a straight geometrical relationship, you can see that the Earth Convergency between the meridians at A and B is equal to the change in Great Circle true direction, as it runs from A to B.

Using the same terms as on the sketch, this may be put up as a mathematical formula:

$$\text{Earth convergency} = GCTT_B - GCTT_A$$

If $GCTT_A$ = 040 ° and $GCTT_B$ = 070°, the formula will give the convergency as -30°. Such a situation is shown in *fig. GN 4.7*. For use in complex navigation formulas the sign of the convergency must be observed. For use in comparatively simple calculations,

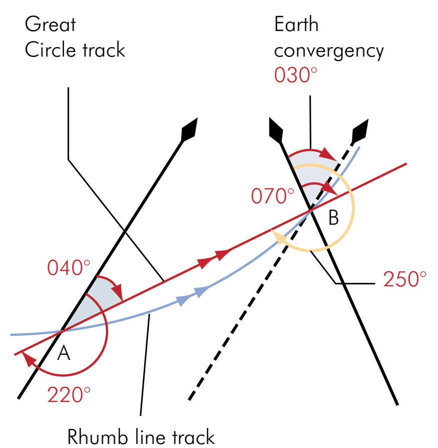


Fig. GN 4.7 Earth Convergency - Northern Hemisphere such as are required for the ATPL exam, the sign may be disregarded. When the sign is disregarded, the problem at hand must always be studied and the answer found by reasoning whether the Great Circle in each case is running towards greater values of direction or towards smaller values. Always use a simple sketch to resolve this problem. Always remembering that the Great Circle always lies on the Polar side of the Rhumb line.

If the reciprocal track on the sketch, B to A, is studied, it will be found that GCTT at B is 070° + 180° = 250°, and GCTT at A is 040° + 180° = 220°.

Using the formula:

$$\text{Earth convergency B - A} = GCTT_B - GCTT_A$$
$$\text{Earth convergency B - A} = 250° - 220°$$
$$\text{Earth convergency B - A} = +30°$$

Earth Convergency may also be found geometrically by transferring the direction of the meridian through the first position parallel to itself through the second position, as shown in *fig GN 4.7* (dotted line). This drawing method should be used to resolve any Earth or Chart Convergency problems.

Summary

The exact value of Earth convergency between two positions is equal to the change in direction of the Great Circle running through these positions. This may be expressed by the equation:

$$\text{Earth convergency} = GCTT_B - GCTT_A$$

4.3 Rhumb Lines

4.3.1 Rhumb Lines

A line, on the surface of the earth, which crosses the meridians at a constant angle is called a Rhumb Line (RL).

An extended Rhumb Line will run from the Equator towards the geographic Poles as a spiral. If the Rhumb Line is extended in the opposite direction from the Equator it will also form a spiral towards the other Pole. A Rhumb Line, passing the Equator on its way to the North Pole, is shown in *fig. GN 4.8.* This is a purely theoretical situation, as in air navigation we are never interested in following a Rhumb Line over such long distances.

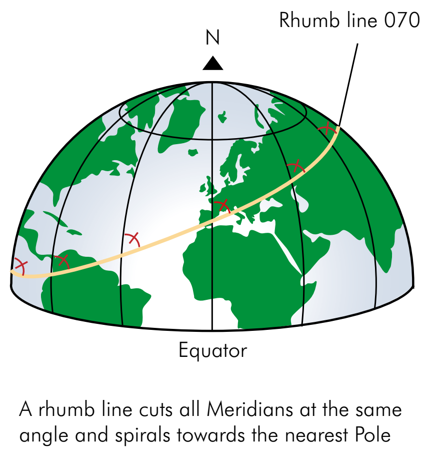


Fig. GN 4.8 Rhumb line

It can mathematically be proven that such a spiral never actually reaches the pole. However this is of no practical interest in air navigation, and we will discuss the Rhumb Line as if it is running as a spiral from Pole to Pole.