4.1 Details on Great Circles

In *fig. GN 4.1* two Great Circle/Rhumb Line cases are shown, one in each hemisphere. In each case the shorter distance between any 2 points will be via the Great Circle route.

When the two points concerned are separated by 180° of longitude, i.e., they lie on a meridian and its associated anti-meridian, e.g. 000°E/W and 180° E/W. In this case the Great Circle will follow the meridian until it reaches the closer pole and then the anti-meridian until reaching the second point. It is easy to calculate this Great Circle distance.

In *table GN 4.1* some figures are given for comparing the Great Circle and the Rhumb Line tracks between place of departure at 60°N 010°E and different destinations, all of which are located on the 60°N parallel. In this case the Rhumb Line track will be 270°, and the Rhumb Line distance is easy to calculate, using the formula for departure.

Observing the Great Circle track and the rhumb line track between the same positions on the Earth’s surface:

- The Great Circle track will continuously change its true direction, while the
rhumb line track by definition has a fixed true direction

- The Great Circle track will be located between the Rhumb Line track and the nearest Pole
- The Great Circle track will be the shortest of the two, and the difference in distance between the two tracks will increase with latitude and with change of longitude between the two positions.

General calculation of great circle directions and distances are today made using handheld electronic calculators, or navigation calculators built into navigation equipment such as RNAV, GPS and INS.

**Fig. GN 4.2** shows a view of the Earth as seen from a point above the Equator. A Great Circle is passing through the position on the Equator directly below the viewer at 100°00’W. The Great Circle has an inclination of 33° as it crosses the Equator, which is an angle of 57° to the meridian. From this perspective it is easy to see that the Great Circle, on its path northeasterly from the Equator, will reach higher latitudes. The highest latitude reached will be the same as the inclination to the Equator i.e. 33°00’N.

It will also be appreciated that the longitude of the Great Circle, where it reaches its highest latitude, will be 90° different from the longitude of the position where it crosses the Equator, in this case 010°00’W. It will then proceed back towards the Equator crossing it 19° removed from the highest point i.e. 080°00’E. The Great Circle then continues to the highest southern latitude of 33°00’S at 170°00’E.

**Vertex**

The position where the Great Circle reaches its highest latitude is called the **vertex** of that Great Circle. **Fig. GN 4.3** shows a Great Circle with its Northern Hemisphere vertex. In order not to complicate the sketch, a broken line indicates the other half of the Great Circle (running on the far side of the Earth).

Every great circle has two vertices; one in the Northern and one in the Southern Hemisphere.

At the vertex the direction of the Great Circle is 090° or 270°, and it is at this point closest to the nearest Pole, and furthest from the Equator. Meridians and the Equator are Great Circles that have special
characteristics. They may be considered to have no vertices, see fig. GN 4.3.

As an example some satellites orbit the Earth in circles. These may be regarded as Great Circles, and the paths of the satellite, projected down onto the Earth’s surface, may also be regarded as great circles. It will therefore be evident that the satellite’s orbit will not reach a higher latitude than the inclination it has with the Equator.

When considering the orbital paths of satellites, it is well to remember that they are following a path in space. A circular orbit will retain its circular shape but, as the Earth rotates under the satellite at the same time, the track of the satellite on the surface of the Earth will not be a Great Circle.

However, the northern and southern limits of the satellite track will still be given by the inclination at the Equator passing.

Summary on Great Circles
- A Great Circle has its vertex where it reaches its highest latitude.
- At the Vertex the Great Circle true direction is 090°/270°, running East – West.
- A Great Circle will have 2 vertices, one on the northern and one on the southern hemisphere.
- The vertices will be located with dlong 090° from the positions where the Great Circle cuts the Equator.
- The latitudes of the vertices will equal the inclination between the plane of Equator and the plane of the Great Circle.

Example
A satellite has an area of coverage on the surface of the Earth that is 600 NM wide. The most northerly position to be covered by this satellite is 72°00’N. What should the inclination with the Equator be?

Solution
The satellite will at any time cover a circle with radius 300 NM on the surface of the Earth. This means that the satellite track may be 300 NM south of its most northerly point of coverage. 300 NM south equals 300 NM/60 = 5° of latitude.
The Vertex must be at 72°00'N - 5° = 67°00'N.

The inclination at Equator must be the same as latitude of the vertex, 67°.

(An inclination of 67° will result in the satellite coverage just touching 72°00'N on every orbit, and the period this latitude is covered will be very short. Better coverage at 72°00'N will be possible by increasing the inclination).

4.2 Earth Convergency

4.2.1 Earth Convergency of the Meridians

Fig. GN 4.4 shows a part of the Northern Hemisphere. If the meridians 01000W and 01000E are considered, it is clear that they both cross the Equator at right angles, and consequently are parallel when they cross the Equator. When these meridians meet, or intercept, at the poles, they intercept with an angle of 20 degrees. This angle is the same value as the dlong between them.

This is an important finding, due to the fact that the Earth is a spherical surface, as we travel between the Equator and the poles, two meridians change directions with an amount dependant on the difference of longitude between them.

In the case above we could say that the convergency of the meridians is zero at the Equator and increasing to 20°, or dlong, at the poles.

The change of the direction we have seen is obviously increasing with increased latitude, but the change is not linear, but a function of the sine of the latitude and dlong.

Therefore we can derive a formula for Earth Convergency:

\[
\text{Earth convergency} = \text{dlong} \times \sin(\text{lat})
\]

\[
= \Delta \lambda \times \sin \varphi
\]

**Definition**

**Earth Convergency** is the angle of inclination of meridians towards one another.

In fig. GN 4.5 the most important facts on Earth Convergency are shown. The
consequences of these facts will be discussed in the chapter on Charts.

**Earth Convergency**
- Is the difference in direction between two meridians
- Increases with an increase in latitude
- Increases with an increase in longitude
- Is zero at the Equator and is equal to dlong at the poles
- May be calculated at any latitude by using the equation:
  
  \[
  \text{Earth convergency} = \text{dlong} \times \sin (\text{lat})
  \]
  
  \[
  = \Delta \lambda \times \sin \phi
  \]

- Earth Convergency between two positions at different latitudes, may be calculated to an approximate value substituting “lat” by “mean lat” in the equation above.

### 4.2.2 Earth Convergency Calculations

A simple formula may be used to calculate the convergency between two meridians:

\[
\text{Earth convergency} = \text{dlong} \times \sin (\text{mean lat})
\]

\[
= \Delta \lambda \times \sin \phi_m
\]

(Only use the above given formula when: \(\Delta \lambda < 90^\circ\) and the distance < 1000 NM.)

As mentioned above, the dlong has to be entered in the formula as degrees and decimals.

The formula is only correct for one value of latitude. If we require to find the Earth Convergency between the meridians running through two positions at different latitude, mean latitude between the positions must be substituted for Lat in the formula. In this case the formula is no longer accurate, and great care should be taken using it. Generally the inaccuracies increase with higher latitudes and higher values of dlong. The reason for this is that sine to an angle is not a linear function, the arithmetic mean value of lat is therefore not correctly used.

The Earth Convergency of meridians may also be evaluated by observing the direction of Great Circles. It is equivalent to the angular change of direction of a great circle course as it passes from one meridian to another.

Using this technique we may find the exact value of Earth Convergency between
two positions even if they are at different latitudes and at different longitudes.

GCTT_A is an abbreviation for “Great Circle True Track Initial”, and GCTT_B stands for “Great Circle True Track Final”. Even if the term “Track” is used in these abbreviations, the short terms are often used also for Great Circles representing types of line other than tracks, even if the correct procedure would then be to use “GCT_T” and “GCT_B”. Other expressions may be encountered.

In fig. GN 4.6 the positions A and B, situated in the Northern Hemisphere, are indicated as well as the meridians running through these positions. The Great Circle running through A and B is shown. Although the diagram may not accurately represent the directions, it does illustrate that the true direction of the Great Circle, as it passes through A, (labelled GCTT_A on the sketch), is of a lower value than the true direction of the Great Circle as it passes B.

The direction at B is labelled GCTT_B. Using a straight geometrical relationship, you can see that the Earth Convergence between the meridians at A and B is equal to the change in Great Circle true direction, as it runs from A to B.

Using the same terms as on the sketch, this may be put up as a mathematical formula:

\[ \text{Earth convergency} = \text{GCTT}_B - \text{GCTT}_A \]

If GCTT_A = 040° and GCTT_B = 070°, the formula will give the convergency as -30°. Such a situation is shown in fig. GN 4.7. For use in complex navigation formulas the sign of the convergency must be observed. For use in comparatively simple calculations,
such as are required for the ATPL exam, the sign may be disregarded. When the sign is disregarded, the problem at hand must always be studied and the answer found by reasoning whether the Great Circle in each case is running towards greater values of direction or towards smaller values. Always use a simple sketch to resolve this problem. Always remembering that the Great Circle always lies on the Polar side of the Rhumb line.

If the reciprocal track on the sketch, B to A, is studied, it will be found that GCTT at B is $070^\circ + 180^\circ = 250^\circ$, and GCTT at A is $040^\circ + 180^\circ = 220^\circ$.

Using the formula:

$$\text{Earth convergency } B - A = GCTT_B - GCTT_A$$

$$\text{Earth convergency } B - A = 250^\circ - 220^\circ = +30^\circ$$

Earth Convergency may also be found geometrically by transferring the direction of the meridian through the first position parallel to itself through the second position, as shown in fig GN 4.7 (dotted line). This drawing method should be used to resolve any Earth or Chart Convergency problems.

**Summary**

The exact value of Earth convergency between two positions is equal to the change in direction of the Great Circle running through these positions. This may be expressed by the equation:

$$\text{Earth convergency } = GCTT_B - GCTT_A$$

It can mathematically be proven that such a spiral never actually reaches the pole. However this is of no practical interest in air navigation, and we will discuss the Rhumb Line as if it is running as a spiral from Pole to Pole.